Entanglement Through the CHSH Game

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A word that is often thrown around in popular science circles on the internet and is a favorite target for many flavors of quantum theories. There have been countless interpretations for what entanglement means, such as Many Worlds, the Copenhagen interpretation, pilot-wave theory, plus plenty more not listed here.

This is typically where I personally see much of the time around entanglement spent: on the "meaning." But the interpretation of it is entirely unnecessary to answer the question that, in my opinion, is the one that really matters:

What even is entanglement?

According to Wikipedia, "Quantum entanglement is the phenomenon where the quantum state of each particle in a group cannot be described independently of the state of the others, even when the particles are separated by a large distance" [1]. This is to say, when a pair of particles is entangled, they share a correlation with each other.

How correlated are they?

More than any local classical model allows.

Before we get into the messy question of how something can be "more correlated than any local classical model allows," we should first define what a classical model does allow.[1] For this, we use the framework of Bell's theorem [2], in the operational form given by Clauser, Horne, Shimony, and Holt [3].

The Game

We will have three participants in our game: Alice, Bob, and Charlie (A, B, and C respectively). Charlie prepares a pair of input bits

$$(x, y) \in \{0, 1\}^2,$$

and sends x to Alice and y to Bob. In this setup, Charlie also sends (x, y) to an AND gate that computes

$$x \wedge y.$$

[1] "Classical" here means two specific assumptions: each player's output depends only on their own input (*locality*), and those outputs have definite values even for inputs that were never sent (*realism*).

Alice and Bob each output a single bit, a and b respectively, which are fed into an XOR gate that computes

$$a \oplus b.$$

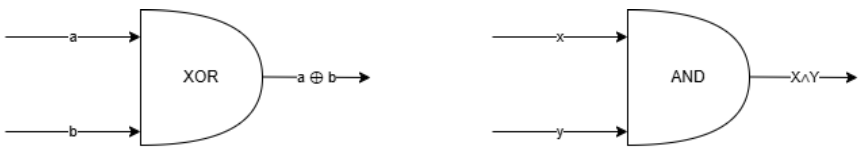
Finally, an XNOR gate checks whether these two results agree. In logical form, the win condition is

$$\neg\big((a \oplus b) \oplus (x \wedge y)\big) = 1,$$

which is equivalent to requiring

$$a \oplus b = x \wedge y.$$

If this condition holds, Alice and Bob score a point. (See Fig. 1 for a diagram of the AND and XOR gates and their truth tables.)



a, b ∈ {0, 1}

XOR			
0	1	0	1
0	0	1	1
0	**1**	**1**	**0**

x, y ∈ {0, 1}

AND			
0	1	0	1
0	0	1	1
0	**0**	**0**	**1**

Figure 1: XOR and AND gates with their truth tables.

Alice and Bob are strictly forbidden from communication during the game. Alice can only react based on x, and Bob only on y. They may meet before the game begins to discuss strategies if they would like.

There are many strategies Alice and Bob could implement, but even with the optimal classical strategy, their probability of winning is bounded. In general we can write the winning probability in the classical case as

$$P_{\text{classical}}(W = 1) = \frac{1}{4} \sum_{x,y \in \{0,1\}} P\big[(a \oplus b) = (x \wedge y)\big], \tag{1}$$

where the factor of $1/4$ comes from the four equally likely input pairs (x, y).

Classical Bound

In the classical case, Alice's response is fully determined by her input bit x, and Bob's by y. For a deterministic strategy we can label their outputs by

$$a_0, a_1 \in \{0, 1\}, \qquad b_0, b_1 \in \{0, 1\},$$

where a_x is Alice's output when she receives x, and b_y is Bob's output when he receives y.[2]

The win condition for the CHSH game is

$$a \oplus b = x \wedge y.$$

For the four possible input pairs (x, y), this yields the four constraints

$$\begin{aligned} (x, y) = (0, 0) &: \quad a_0 \oplus b_0 = 0, \\ (x, y) = (0, 1) &: \quad a_0 \oplus b_1 = 0, \\ (x, y) = (1, 0) &: \quad a_1 \oplus b_0 = 0, \\ (x, y) = (1, 1) &: \quad a_1 \oplus b_1 = 1. \end{aligned}$$

Now take the XOR of all four left-hand sides:

$$(a_0 \oplus b_0) \oplus (a_0 \oplus b_1) \oplus (a_1 \oplus b_0) \oplus (a_1 \oplus b_1).$$

Each of a_0, a_1, b_0, b_1 appears exactly twice, and since $z \oplus z = 0$ for any bit z, the entire expression reduces to

$$(a_0 \oplus a_0) \;\oplus\; (a_1 \oplus a_1) \;\oplus\; (b_0 \oplus b_0) \;\oplus\; (b_1 \oplus b_1) \;=\; 0.$$

On the right-hand side, we have

$$0 \oplus 0 \oplus 0 \oplus 1 = 1.$$

Thus, assuming that all four constraints hold simultaneously leads to

$$0 = 1,$$

a contradiction. Therefore, no deterministic classical strategy can satisfy all four CHSH constraints at once. The best any deterministic strategy can do is to satisfy three of the four input cases, giving a success probability

$$P^{\max}_{\text{classical}}(W = 1) = \frac{3}{4} = 0.75. \tag{2}$$

Any randomized classical strategy is just a probability distribution over deterministic ones, so it cannot exceed this optimal value. No purely classical model can do better.

Figure 2 shows the full classical CHSH game setup in logic-gate form.

[2] Both classical assumptions from the introduction are encoded in this notation: a_x depends only on Alice's own input (locality), and all four values a_0, a_1, b_0, b_1 are treated as simultaneously well-defined even though only one input pair is ever sent (realism).

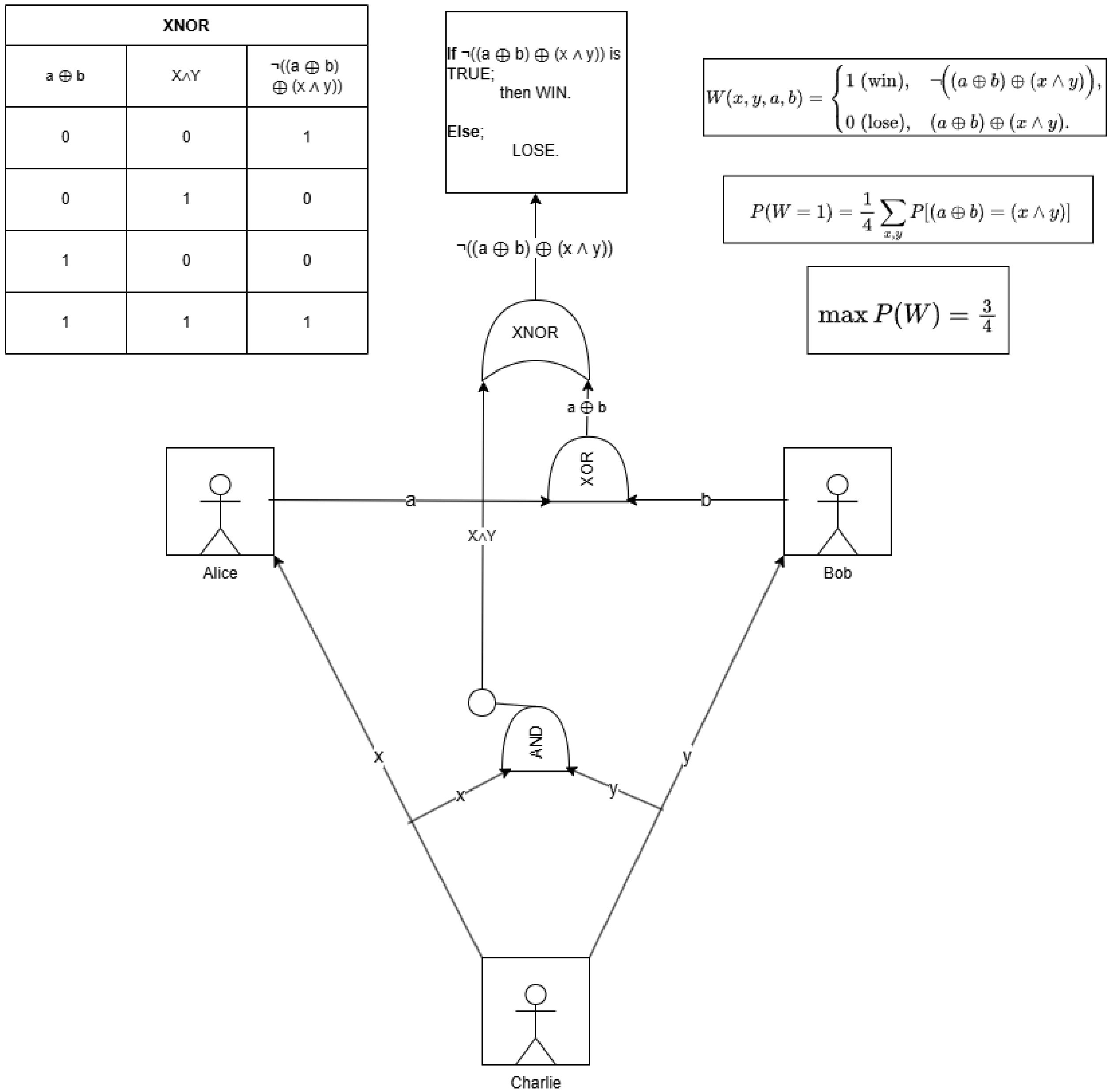


Figure 2: Classical CHSH game represented with logic gates. Alice and Bob receive inputs x and y, output bits a and b, and win when $a \oplus b = x \wedge y$.

The Quantum Strategy

As established above, with an optimal classical strategy Alice and Bob are only able to win the game at most 75% of the time. But what about a quantum strategy? This is where things get interesting.

In the quantum version, Charlie still prepares inputs (x, y) just as before, but he also prepares an entangled pair of qubits in the Bell state

$$|\psi_{ab}\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle_a |0\rangle_b + |1\rangle_a |1\rangle_b\big), \tag{3}$$

sending one particle to Alice and the other to Bob along with x and y, respectively.

We also add two measurement devices (shown as boxes with diagonal lines in Fig. 3) that measure the state of the incoming qubit. Alice's detector angle is determined by her input x and Bob's by his input y. We can denote these measurement settings by

$$\varphi_{A,x} \quad \text{and} \quad \varphi_{B,y},$$

and the detector outputs define their bits a and b as measurement outcomes:

$$a \sim \mathcal{M}_A(\varphi_{A,x}, |\psi_{ab}\rangle), \tag{4}$$

$$b \sim \mathcal{M}_B(\varphi_{B,y}, |\psi_{ab}\rangle). \tag{5}$$

The rest of the rules of the game are exactly the same as they were in the classical version: they still win only when $a \oplus b = x \wedge y$.

Given these changes, the probability that Alice and Bob win a round in the quantum case can be written as

$$P_q(W = 1) = \frac{1}{8}\sum_{x,y}\left[1 + (-1)^{x\wedge y}\cos(\varphi_{A,x} - \varphi_{B,y})\right]. \tag{6}$$

Quantum Bound

Starting from Eq. (6), define the correlators

$$E_{xy} = \cos(\varphi_{A,x} - \varphi_{B,y}),$$

so that each term in the sum becomes

$$\frac{1}{2}\left[1 + (-1)^{x\wedge y}E_{xy}\right].$$

Substituting this expression back into the average over the four input pairs,

$$P_q(W = 1) = \frac{1}{8}\left(4 + E_{00} + E_{01} + E_{10} - E_{11}\right).$$

The combination

$$S = E_{00} + E_{01} + E_{10} - E_{11}$$

is exactly the CHSH correlator. Thus the quantum success probability can be written compactly as

$$P_q(W = 1) = \frac{1}{2} + \frac{S}{8}.$$

Tsirelson's theorem [4] states that any quantum strategy must satisfy the bound

$$|S| \le 2\sqrt{2}.$$

Applying this to the expression above gives the maximum achievable quantum value, known as Tsirelson's bound:

$$P_q^{\max}(W = 1) = \frac{1}{2} + \frac{2\sqrt{2}}{8} = \frac{2+\sqrt{2}}{4} \approx 0.8536. \tag{7}$$

This bound is saturated by the measurement angles

$$\varphi_{A,0} = 0^\circ, \quad \varphi_{A,1} = 90^\circ, \quad \varphi_{B,0} = 45^\circ, \quad \varphi_{B,1} = -45^\circ,$$

which give $E_{00} = E_{01} = E_{10} = \frac{\sqrt{2}}{2}$ and $E_{11} = -\frac{\sqrt{2}}{2}$, hence $S = 2\sqrt{2}$. This decisively violates the limit set by the optimal classical strategy in Eq. (2).

Figure 3 shows the quantum version of the CHSH game, with the entangled source and measurement settings made explicit.

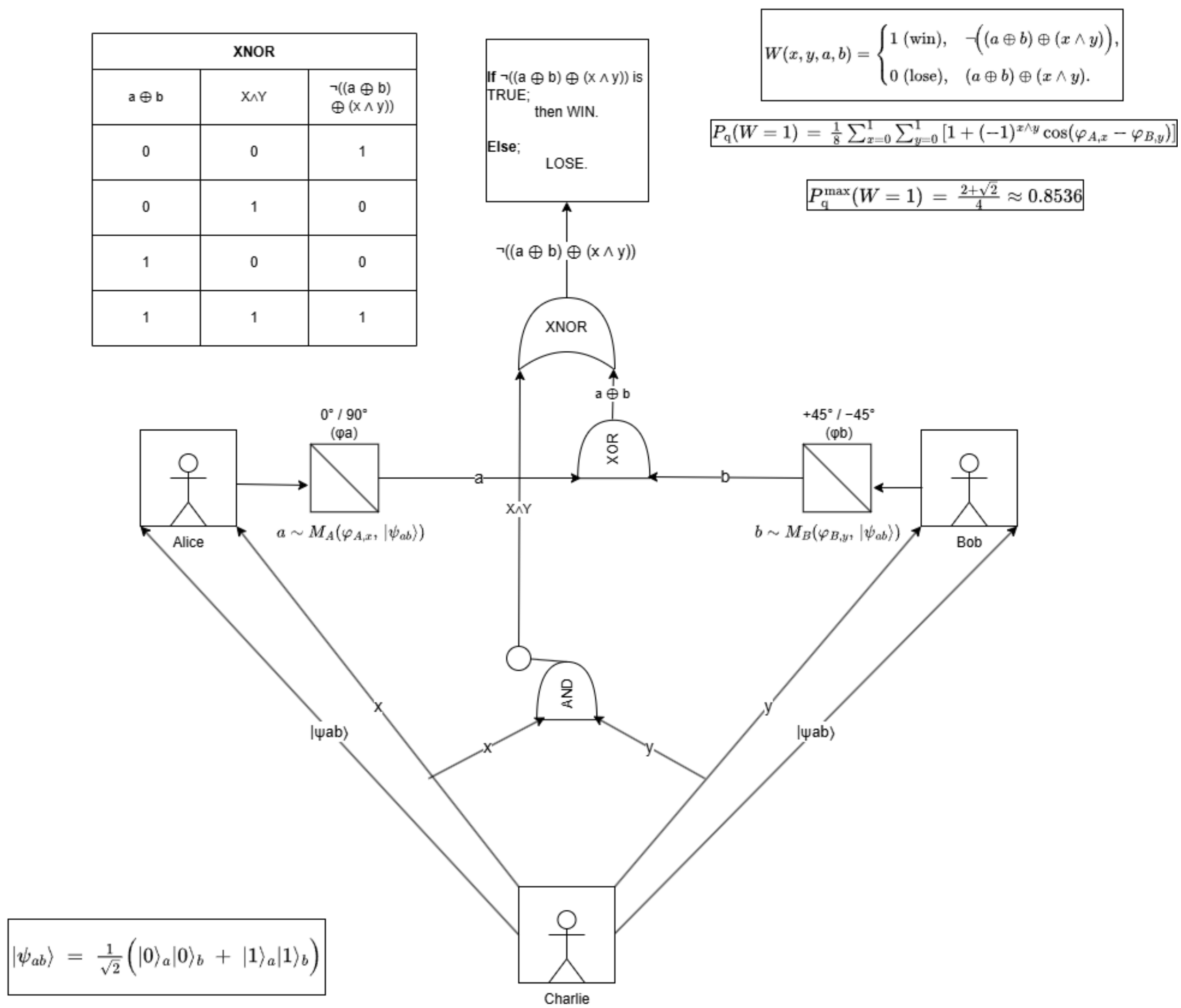


Figure 3: Quantum CHSH game. Alice and Bob share an entangled state $|\psi_{ab}\rangle$ and perform measurements at angles determined by x and y. The win condition remains $a \oplus b = x \wedge y$, but the achievable success probability is higher than in any local classical model.

Conclusion

$$\boxed{\Delta_{\text{corr}} = P_q^{\max} - P_{\text{classical}}^{\max} \approx 0.1036}$$

The roughly ten percent gap between the classical limit (75%) and the quantum bound (about 85%) shows that quantum mechanics predicts fundamentally stronger correlations than any local classical model can achieve — and experiment confirms that nature actually delivers them. Violations of Bell inequalities have been observed repeatedly, beginning with Freedman and Clauser [5] and Aspect, Dalibard, and Roger [6], and culminating in loophole-free tests such as Hensen *et al.* [7]. The 2022 Nobel Prize in Physics was awarded to Clauser, Aspect, and Zeilinger for this line of experimental work [8].

A link to this PDF and a simulation that lets you see the game played out in real time are included below. Go try it out at:

`https://just-some-vibe-physics.netlify.app/tools/bells-game`

References

[1] "Quantum entanglement," *Wikipedia*, `https://en.wikipedia.org/wiki/Quantum_entanglement`.

[2] J. S. Bell, "On the Einstein Podolsky Rosen paradox," *Physics Physique Fizika* **1**, 195–200 (1964).

[3] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, "Proposed experiment to test local hidden-variable theories," *Phys. Rev. Lett.* **23**, 880–884 (1969).

[4] B. S. Cirel'son, "Quantum generalizations of Bell's inequality," *Lett. Math. Phys.* **4**, 93–100 (1980).

[5] S. J. Freedman and J. F. Clauser, "Experimental test of local hidden-variable theories," *Phys. Rev. Lett.* **28**, 938–941 (1972).

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[7] B. Hensen *et al.*, "Loophole-free Bell inequality violation using electron spins separated by 1.3 kilometres," *Nature* **526**, 682–686 (2015).

[8] "The Nobel Prize in Physics 2022," *NobelPrize.org*, `https://www.nobelprize.org/prizes/physics/2022/summary/`.